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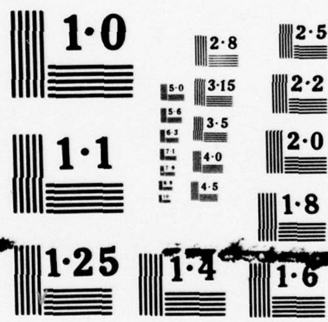

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Task No. NR064-576

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TIME SERIES DETERMINATION OF TRANSFER FUNCTIONS IN RANDOM FATIGUE.

10 T. C. HUANG, VINOD K. NAGPAL and K. S. SHEN

Department of Engineering Mechanics
University of Wisconsin—Madison

15 NØØØ14-76-C-Ø825

Project: RANDOM FATIGUE

Technical Report No. 14 UW/RF-2

11 Dec[redacted]77 12 46p.

Department of Engineering Mechanics

College of Engineering

University of Wisconsin—Madison

Madison, Wisconsin

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TIME SERIES DETERMINATION OF TRANSFER FUNCTIONS
IN RANDOM FATIGUE

T. C. Huang, Vinod Nagpal and K. S. Shen
Department of Engineering Mechanics
University of Wisconsin-Madison
Madison, Wisconsin 53706

Abstract

Time series determination of the transfer function which relates the input random excitation and the output response in random fatigue experiment is established. This process involves determination of univariate time series of input and output, transfer function and noise models, and the transfer function-noise model.

INTRODUCTION

In the investigation of random fatigue and design of random fatigue experiments a transfer function which relates the input random excitation and the output random response is involved. Time series technique has been chosen for the transfer function estimation, which requires first, the identification of univariate time series models for the random excitation and the random response, and second, the estimation of the transfer function-noise model.

There are three different types of univariate time series models, namely, autoregressive, moving average, and mixed models. Estimates and plot of autocorrelations and partial autocorrelations of the digital signal are used for the identification and initial estimation of parameters of a univariate time series model. Final estimation of parameters is done by regression analysis. Adequacy of the model is checked by the following tests on residuals: (a) test of autocorrelations at all lags, and (b) χ-square test on sum of square of residuals.

The transfer function-noise model consists of two parts, transfer function and noise. Impulse response function is obtained from the estimates of cross-correlations between the digital input and digitized response. The order and initial estimates of parameters of the transfer function are obtained from impulse response weights. These initial estimates are used to identify the univariate time series model for noise (and in the same operation improved estimates of parameters of the transfer function are obtained). Initial estimates of parameters of the noise model are obtained as in univariate time series models. Estimates of parameters of the transfer function-noise model are obtained by using the estimates

of transfer function and noise model parameters. Adequacy of the transfer function-noise model is checked by following tests on residuals: (a) tests of autocorrelations and partial autocorrelations at all lags, and (b) χ-square test on the sum of squares of autocorrelations and cross-correlations.

A single reference [1] is involved in this report. Particular pages are referred to whenever necessary.

I. SIGNAL

The input signal, the white noise, was generated from normally distributed random numbers. This digital signal was recorded on the tape and was converted to an analog signal by a D to A converter. The analog signal was transmitted to the shaker in the vibration lab. The response signal of the specimen mounted on the shaker was transmitted to the A to D converter, digitized and recorded on the same tape on which the digital input signal was recorded. The sampling interval of digitization was chosen at 2.60 millisec., which is the minimum sampling interval capability of the machine. The same sampling interval was used to convert the random numbers to the analog input signal. Several complete runs were tried to assure that the system would perform properly. One of the inputs and its corresponding response was arbitarily chosen for transfer function analysis.

II. UNIVARIATE TIME SERIES MODELS - BASICS

1. Model

A univariate time series model of order (p,q) is expressed as

$$x_t - \phi_1 x_{t-1} - \phi_2 x_{t-2} - \cdots - \phi_p x_{t-p} = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \cdots - \theta_q a_{t-q} \quad (1)$$

or

$$(1-\phi_1B-\phi_2B^2 - \dots - \phi_pB^p)x_t = (1-\theta_1B-\theta_2B^2 - \dots - \theta_qB^q)a_t \quad (2)$$

where B is backshift operator ($Bx_t = x_{t-1}$), x_t is the observation at any time t, a_t is white noise at any time t, and ϕ and θ are parameters of autoregressive and moving average, respectively.

The above equation can be written symbolically as

$$\phi(B)x_t = \theta(B)a_t$$

in which

$$\phi(B) = 1 - \phi_1B - \phi_2B^2 - \dots - \phi_pB^p$$

$$\theta(B) = 1 - \theta_1B - \theta_2B^2 - \dots - \theta_qB^q$$

For a stationarity condition of the process, the roots of the characteristic equation $\phi(B) = 0$ must lie outside the unit circle. Similarly, the roots of $\theta(B) = 0$ must lie outside the unit circle if the process is to be invertible.

In general the univariate time series model is represented by ARIMA (p,d,q). AR stands for autoregressive, I for integrated and MA for moving average; p in the parentheses stands for the order of autoregressive, d for the order of differencing and q for the order of moving average. The sum of p and q gives the total number of parameters in the model. There are three special cases of ARIMA models, AR(p), MA(q) and ARMA(p,q) as simplified forms of ARIMA(p,0,0), ARIMA(0,0,q) and ARIMA(p,0,q), respectively.

2. Identification of Model

The type and order of a model is identified by the shapes of autocorrelations (ACF) and partial autocorrelations (PACF). When ACF and PACF are plotted, there are two basic groups of shapes, the damped out group and the cutoff group as listed in Tables 1 and 2. Fig. 1 shows

the decaying expononttal, the damped sine wave, and the mixture of two decaying exponentials; Fig. 2 shows cutoffs after lag 1 and after lag 2. For the complex shapes of ACF and PACF, the models and orders will have to be guessed. The guessed model would be a higher order of AR or MA. The only hint of the guess may be obtained from the physical system involved as each dominant natural frequency of the system indicates an order 2 of AR.

3. Initial Estimates of Parameters

Once the order of the model is identified the number of parameters is known. Initial estimates of parameters is computed using the magnitudes of autocorrelations. The equations used to compute the initial estimates are derived [2] as follows.

Let the univariate model be expressed as equation (1). Premultiplying by x_{t-k} and taking expectations give

$$E[x_{t-k}x_t] - \phi_1 E[x_{t-k}x_{t-1}] - \dots - \phi_p E[x_{t-k}x_{t-p}]$$

$$= E[x_{t-k}a_t] - \theta_1 E[x_{t-k}a_{t-1}] - \dots - \theta_q E[x_{t-k}a_{t-q}]$$

or

$$\gamma(k) - \phi_1\gamma(k-1) - \dots - \phi_p\gamma(k-p) = \gamma_{xa}(k) - \theta_1\gamma_{xa}(k-1) - \dots - \theta_q\gamma_{xa}(k-q) \qquad (3)$$

where $\gamma(k)$ is the auto-covariance at lag k and $\gamma_{xa}(k)$ is the cross covariance at lag k. An investigation of $\gamma_{xa}(k)$ for various values of lags shows

$$\gamma_{xa}(k) = 0 \qquad k > 0$$

$$\gamma_{xa}(0) = \sigma_a^2 \qquad k = 0$$

$$\gamma_{xa}(k) \neq 0 \qquad k < 0$$

By definition

$$\gamma(0) = \sigma_x^2$$

Note also that σ_x^2 can be expressed in terms of σ_a^2 multiplied by a function of ϕ and θ.

Depending on the values of lag k, the estimation of parameters ϕ and θ falls into the following two cases.

Case 1. $k \geq q + 1$ In this case all γ_{xa} on the right side of equation (3) vanish. Therefore

$$\gamma(k) - \phi_1\gamma(k-1) - \phi_2\gamma(k-2) - \ldots = 0$$

The autocorrelation at lag k is $\rho(k) = \gamma(k)/\gamma(0)$ which implies $\rho(0) = 1$. Then the above equation, after dividing by $\gamma(0)$, becomes

$$\rho(k) - \phi_1\rho(k-1) - \phi_2\rho(k-2) - \ldots = 0 \tag{4}$$

from which the parameters ϕ can be solved.

Case 2. $k \leq q$ In this case the cross-covariances $\gamma_{xa}(-1)$, $\gamma_{xa}(-2)$, ..., $\gamma_{xa}(-q)$ will have to be evaluated successively. To evaluate $\gamma_{xa}(-1)$, postmultiply equation (1) by a_{t-1} and take expectations. This results in

$$E[x_t a_{t-1} - \phi_1 x_{t-1} a_{t-1} - \ldots - \phi_p x_{t-p} a_{t-1}]$$

$$= E[a_t a_{t-1} - \theta_1 a_{t-1} a_{t-1} - \ldots - \theta_q a_{t-q} a_{t-1}]$$

from which

$$\gamma_{xa}(-1) - \phi_1\sigma_a^2 = -\theta_1\sigma_a^2$$

or

$$\gamma_{xa}(-1) = (\phi_1 - \theta_1)\sigma_a^2 \tag{5}$$

This procedure can be continued to obtain $\gamma_{xa}(-2)$, $\gamma_{xa}(-3)$, ..., $\gamma_{xa}(-q)$ successively. Now $\gamma(0)$ and all γ_{xa} are expressed in terms of functions of ϕ and θ multiplied by σ_a^2. Dividing equation (3) by $\gamma(0)$ gives

$$\rho(k) - \phi_1\rho(k-1) - \dots - \phi_p\rho(k-p)$$

$$= \frac{\gamma_{xa}(k)}{\gamma(0)} - \theta_1 \frac{\gamma_{xa}(k-1)}{\gamma(0)} - \dots - \theta_q \frac{\gamma_{xa}(k-q)}{\gamma(0)} \tag{6}$$

In the above equation all ρ are known and all the terms of $\gamma_{xa}/\gamma(0)$ are expressed in terms of ϕ and θ, σ_a^2 being cancelled. In general this equation is nonlinear, therefore ϕ and θ will have to be solved by an approximation method. Procedures to estimate ϕ and θ for a simple ARMA(1,1) model is illustrated in [3].

4. Final Estimates of Parameters

From the initial estimates of ϕ and θ parameters, estimates of observation, residuals and sum of squares of residuals can be computed. Final estimates of ϕ and θ parameters are obtained by regression analysis based on minimizing the sum of squares of residuals.

5. Diagnostic Checking

In order to check the adequacy of the model which has been identified and estimated, diagnostic checking is required. For a univariate model, diagnostic checking consists of following two checks:

a. ACF and PACF checks The autocorrelations and partial autocorrelations of the residuals at all lags should be statistically insignificant, i.e. they should be less than two standard deviations.

b. χ^2-test The χ^2 value is computed as follows:

$$\chi_d^2 = n \sum_{i=1}^{m} r_i^2(\hat{a}) \tag{7}$$

where d is the degree of freedom, n is the number of observations, $r_i(\hat{a})$ is the autocorrelation of estimated residuals $\hat{a}$ at lag i, and m is the number of autocorrelations used. d is related to m as

$$d = m - p - q \tag{8}$$

The χ^2 value should be less than the value obtained from the χ^2 table for the same degree of freedom.

The fitted model which meets the above two diagnostic checkings is considered adequate. Otherwise the whole process should be repeated, i.e., to reidentify the model and to estimate its parameters.

For two adequate models, the one which has less number of parameters is preferred.

III. TRANSFER FUNCTION-NOISE MODEL - BASICS

1. Models

A transfer function model of the order (r,s) in the form of a difference equation is expressed as

$$Y_t - \delta_1 Y_{t-1} - \delta_2 Y_{t-2} - \ldots - \delta_r Y_{t-r} = \omega_o X_{t-b} - \omega_1 X_{t-b-1} - \omega_2 X_{t-b-2} - \ldots - \omega_s X_{t-b-s} \tag{9}$$

in which X_t and Y_t are deviations from the equilibrium of the system input and response, δ and ω are the transfer function parameters, and b is the lag factor. The above equation can be simply written as

$$(1 - \delta_1 B - \ldots - \delta_r B^r)Y_t = (\omega_0 - \omega_1 B - \ldots - \omega_s B^s)X_{t-b} \tag{10}$$

or simply as

$$Y_t = \delta^{-1}(B)\omega(B)X_{t-B}$$

where B is the back shift operator ($BY_t = Y_{t-1}$), and

$$\delta(B) = 1 - \delta_1 B - \delta_2 B^2 - \ldots - \delta_r B^r$$

$$\omega(B) = \omega_0 - \omega_1 B - \omega_2 B^2 - \ldots - \omega_s B^s$$

In practice the system will be infected by disturbances or noise whose net effect is to corrupt the response predicted by the transfer function model by an amount N_t. The combined transfer function-noise model are then written as

$$Y_t = \delta^{-1}(B)\omega(B)X_{t-b} + N_t \tag{11}$$

The noise N_t can be further modeled as a univariate time series model ARIMA (p,d,q),

$$(1-\phi_1 B-\phi_2 B^2-\ldots-\phi_p B^p)N_t = (1-\theta_1 B-\theta_2 B^2-\ldots-\theta_q B^q)a_t \tag{12}$$

or

$$\phi(B)N_t = \theta(B)a_t$$

The order of combined transfer function-noise model is usually represented by (r,s,b). The total number of parameters in the combined model is the sum of r,s,p and q.

2. Transfer Function Model

The difference equation of the transfer function of a discrete dynamic system may be written in the form which corresponds to the convolution integral, as

$$Y_t = v(B)X_t \tag{13}$$

in which the impulse response weights $v(B)$ can be expanded in the form

$$v(B) = v_0 + v_1 B + v_2 B^2 + v_3 B^3 + \ldots$$

Substituting $Y_t = v(B)X_t$ with $v(B)$ in expanded form into equation (10) and equating the coefficients of X_t, we obtain

$$(1 - \delta_1 B - \ldots - \delta_r B^r)(v_0 + v_1 B + v_2 B^2 + \ldots)$$
$$= (\omega_0 - \omega_1 B - \ldots - \omega_s B^s) B^b$$

On equating coefficients of B, we find the following four sets of equations [4]:

$$\begin{aligned}
v_j &= 0 && j < b \\
v_j &= \delta_1 v_{j-1} + \delta_2 v_{j-2} + \ldots + \delta_r v_{j-r} + \omega_0 && j = b \\
v_j &= \delta_1 v_{j-1} + \delta_2 v_{j-2} + \ldots + \delta_r v_{j-r} - \omega_{j-b} && j = b+1,\ b+2,\ \ldots,\ b+s \\
v_j &= \delta_1 v_{j-1} + \delta_2 v_{j-2} + \ldots + \delta_r v_{j-r} && j > b + s
\end{aligned} \tag{14}$$

Therefore the impulse response weights can be divided into four groups as follows.

Group	Impulse Response Weight	Number
1	$v_0, v_1, v_2, \ldots, v_{b-1}$	b
2	v_b	1
3	$v_{b+1}, v_{b+2}, \ldots, v_{b+s}$	s
4	$v_{b+s+1}, v_{b+s+2}, \ldots$	$\geq r$

The parameters δ and ω can be estimated by the use of the four sets of equations (14) provided all the impulse response weights are known and the model (r,s,b) is identified. A minimum number of (b+1+s+r) known impulse response weights is needed in the estimation.

a. <u>Identification</u> In order to identify the model, i.e., to obtain the values of r, s and b, it is necessary to compute the impulse response weights: $v_0, v_1, v_2, \ldots, v_k, \ldots$ and plot v vs k, k being the lag. The impulse response weight with lag k is computed by

$$v_k = \frac{\rho_{\alpha\beta}(k)\sigma_\beta}{\sigma_\alpha} \tag{15}$$

where $\rho_{\alpha\beta}(k)$ is the cross-correlation between input and output at lag k and σ_α, σ_β are the standard deviations of input and output, respectively, provided the input is white noise. Otherwise the prewhitened input and output should be used.

The first set of equations (14) indicates that there are initially b number of zero values of impulse response weights, i.e., $v_0, v_1, \ldots, v_{b-1}$ are zero. From the v-k plot the value of b can be obtained by counting the number of initial zero weights.

The third set of equations (14) indicates that there are $s-r+1$ number of impulse response weights, i.e., $v_b, v_{b+1}, \ldots, v_{b+s-r+1}$ which follow no fixed pattern in the v-k plot. Let this number be n then

$$s - r + 1 = n$$

n is now being counted on the v-k plot between v_{b-1}, which is the last zero value of v, and the first v, which starts the pattern and is usually the highest v. There is no such n when $s < r$.

The fourth set of equations (14) indicates that the values v_j with $j \geq b + s - r + 1$ follow the pattern dictated by the rth order difference equation which has r starting values $v_{b+s}, v_{b+s-1}, \ldots, v_{b+s-r+1}$. For example $r = 1$ for decaying exponential, $r = 2$ for dampled sine wave. For high order difference equation ($r > 2$) the pattern becomes complex and it is difficult to identify both n and r.

For known n and r, s can be computed from

$$s = n + r - 1 \tag{16}$$

The order (r,s,b) is now completely identified. When n and r can not be identified from the pattern, r and s will have to be guessed.

Depending on the types of input the guidance for the guess is described as follows.

Case 1. Input-White noise. Let the input be white noise

$$X_t = a_t$$

The output in univariate and transfer function models is expressed as

$$Y_t = \phi^{-1}(B)\theta(B)a_t = \delta^{-1}(B)\omega(B)X_t \tag{17}$$

The orders of $\phi(B)$ and $\theta(B)$ are p and q while the orders of $\delta(B)$ and $\omega(B)$ are r and s, respectively. As a first guess, $r = p$ and $s = q$.

Case 2. Input-not white noise. If the input is not white noise the univariate models for input and output are expressed as

$$\phi_a(B)X_t = \theta_a(B)a_t$$

$$\phi_b(B)Y_t = \theta_b(B)a'_t$$

When both input and output are prewhitened

$$\theta_a^{-1}(B)\phi_a(B)X_t = a_t$$

$$\theta_b^{-1}(B)\phi_b(B)Y_t = a'_t$$

For the purpose of guess, assume $a_t \simeq a'_t$. Then

$$\theta_b^{-1}(B)\phi_b(B)Y_t = \theta_a^{-1}(B)\phi_a(B)X_t$$

Now the output can be expressed in both univariate and transfer function models as

$$Y_t = \phi_b^{-1}(B)\theta_a^{-1}(B)\phi_a(B)\theta_b(B)X_t = \delta^{-1}(B)\omega(B)X_t \tag{18}$$

The orders of $\phi_b(B)$ and $\theta_a(B)$ are p_b and q_a, the order of $\phi_a(B)$ and $\theta_b(B)$ are p_a and q_b, and the order of $\delta(B)$ and $\omega(B)$ are r and s. As a first guess, $r = p_b + q_a$ and $s = p_a + q_b$.

b. <u>Estimation</u> From 2r number of v: $v_{b+s-r+1}$ $v_{b+s-r+2}$, ..., v_{b+s+r} and r number of the fourth set of equations (14):

$$v_{b+s+1} = \delta_1 v_{b+s} + \delta_2 v_{b+s-1} + \ldots + \delta_r v_{b+s-r+1}$$

$$v_{b+s+2} = \delta_1 v_{b+s+1} + \delta_2 v_{b+s} + \ldots + \delta_r v_{b+s-r+2}$$

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$$v_{b+s+r} = \delta_1 v_{b+s+r+1} + \delta_2 v_{p+s+r-2} + \ldots + \delta_r v_{b+s}$$

δ_1, δ_2, ..., δ_r can be solved.

The first set of equations (14) gives $v_j = 0$, if $j < b$. Then the second equation of equations (14) gives

$$\omega_0 = v_b$$

From (s+1) number of v: v_b, v_{b+1}, ..., v_{b+s}, r number of δ: δ_1, δ_2, ..., δ_r, and s number of third set of equations (14)

$$v_{b+1} = \delta_1 v_b - \omega_1$$

$$v_{b+2} = \delta_1 v_{b+1} + \delta_2 v_b - \omega_2$$

$$v_{b+3} = \delta_1 v_{b+2} + \delta_2 v_{b+1} + \delta_3 v_b - \omega_3$$

. . . .

$$v_{b+s} = \delta_1 v_{b+s-1} + \delta_2 v_{b+s-2} + \ldots + \delta_s v_b - \omega_s \quad (s \leq r)$$

or

$$v_{b+s} = \delta_1 v_{b+s-1} + \delta_2 v_{b+s-2} + \ldots + \delta_s v_b + \ldots + \delta_r v_{b+s-r} - \omega_s \quad (s > r)$$

ω_1, ω_2, ..., ω_s can be obtained.

3. <u>Noise Model</u>

From the transfer function-noise model we have

$$N_t = \delta(B)Y_t - \omega(B)X_{t-b} \qquad (19)$$

This equation can be used to generate a noise series with input and response data series and estimates of transfer function parameters. Using the procedure described previously the univariate time series model for this noise series can be identified and the noise model parameters can be estimated.

4. Transfer Function-Noise Model

The initial estimates of the parameters of the transfer function model and the noise model are evaluated independently for each. These estimates are used to compute the final estimates of all the parameters of transfer function-noise model by regression analysis based on minimizing the sum of squares of residuals.

5. Diagnostic Checking

The diagnostic checking required to ascertain the adequacy of transfer function-noise model consists of four checks as follows.

a. ACF and PACF Checks The autocorrelations and partial autocorrelations of the residuals at all lags should be statistically insignificant, i.e., they should be less than two standard deviations.

b. χ^2-test The χ^2 value of autocorrelations of estimated residuals, evaluated by equation (7), should be less than the value obtained from the χ^2 table for the same degree of freedom.

c. Cross-correlation check The cross-correlations of the estimated residuals and prewhitened input at all lags should be statistically insignificant.

d. χ^2-test The χ^2 value of cross-correlation is computed as follows:

$$\chi_d^2 = n \sum_{i=0}^{m} r_i^2(\alpha\hat{a}) \qquad (20)$$

in which d is the degree of freedom, n is the number of observation, $r_i(\alpha\hat{a})$ is the cross-correlation of prewhitened input α and estimated residuals $\hat{a}$ at lag i, and (m+1) is the number of cross-correlations used. d is related to m as

$$d = (m+1) - (r+s+1)$$

or

$$d = m-r-s \tag{21}$$

The χ^2 value of cross-correlations should be less than the value obtained from the χ^2 table for the same degree of freedom.

IV. UNIVARIATE TIME SERIES MODEL - APPLICATION

1. Input Model

The input signal generated from normally distributed random numbers is white noise. The input series data, 496 in number, are given in Table 3, and their plot is shown in Fig. 3.

a. Identification The autocorrelations of the input series up to 24 lags and their corresponding standard errors are given in Table 4. The autocorrelations are also plotted as shown in Fig. 4. Partial autocorrelations also estimated up to 24 lags are shown in Table 5 and plotted in Fig. 5. The standard error for all the partial autocorrelations is approximated as $1/\sqrt{n}$ where n is the number of observations. In this case, the number of observations is 496, therefore the standard error is approximately .05.

It can be seen that autocorrelations and partial autocorrelations at all 24 lags are statistically insignificant (less than two standard errors); consequently, there is no particular shape visible in either plot of autocorrelations or partial autocorrelation. This implies that the input has neither ϕ nor θ parameters in the model. In other words, the input is

white-noise as it should be.

b. Estimation of parameters As the input series is white noise and has no parameters in the model, the question of estimation of parameters does not arise. Therefore, the model for the input series is

$$x_t = a_t$$

c. Diagnostic checking To assure the adequacy of the model the following two checks were performed:

(1) The autocorrelations and partial autocorrelations given in Table 4 and Table 5 and their corresponding plots in Fig. 4 and Fig. 5 were observed to be statistically insignificant at all 24 lags.

(2) The χ^2 value based on 24 autocorrelations is 28.0 for 24 degrees of freedom and χ^2 value from the χ^2-table with 24 degrees of freedom at .025 level is 39.4.

The above two checks indicate the fitted model is adequate.

2. Output Model

The digitized response which consists of 496 observations is given in Table 6 and plotted in Fig. 6.

a. Identification The autocorrelations of the response series estimated up to 48 lags and their corresponding standard errors are shown in Table 7. The autocorrelations are also plotted in Fig. 7. Partial autocorrelations are also estimated up to 48 lags and the results are shown in Table 8 and plotted in Fig. 8. The standard error of partial autocorrelations is approximated as $1/\sqrt{n}$ where n is the number of observations. In this case the number of observations is 496, therefore the standard error is approximately .05.

It can be seen in Fig. 7 that the autocorrelations are a mixture of exponential and sinusoidal decay and in Fig. 8, that six partial

autocorrelations are nonzero. Assuming that the first two natural frequencies of the system are dominant, the model was guessed to be at least 4th order autoregressive or high order mixed models. AR models of order 4 and higher, and ARMA models of order (3,2) were tried.

b. Initial estimates of parameters AR(4) model was tried first and was found to be inadequate. AR(5) is a good fit but does not meet the requirement of χ^2-test. AR(6) was found to be an adequate fit. On the other hand ARMA(3,2) was also found to be an adequate model. Finally ARMA(3,2) was preferred over AR(6) because it has less number of parameters.

The initial estimates of parameters for the ARMA(3,2) model can be computed from the equations derived from equations (4) and (6) as follows:

$$\rho_3 = \phi_1\rho_2 + \phi_2\rho_1 + \phi_3$$

$$\rho_4 = \phi_1\rho_3 + \phi_2\rho_2 + \phi_3\rho_1$$

$$\rho_5 = \phi_1\rho_4 + \phi_2\rho_3 + \phi_3\rho_2$$

$$\rho_2 = \phi_1\rho_1 + \phi_2 + \phi_3\rho_1 - \theta_2\sigma_a^2/\gamma_0$$

$$\rho_1 = \phi_1 + \phi_2\rho_1 + \phi_3\rho_2 - (\theta_1+\theta_2\phi_2+\theta_1\theta_2)\sigma_a^2/\gamma_0$$

where

$$\gamma_o = \frac{\Omega_1 - \Omega_2\mu}{\Omega_3 - \Omega_4\lambda}\sigma_a^2$$

and

$$\Omega_1 = 1 - \theta_1(\phi_1-\theta_1) - \theta_2(\phi_2-\theta_2-\phi_1^2+\phi_1\theta_1) + \phi_2\theta_2 - \phi_1\phi_2\theta_3$$

$$\Omega_2 = \phi_1 + \phi_2(\phi_1+\phi_3) + \phi_1\phi_3(\phi_1+\phi_3) + \phi_2\phi_3$$

$$\Omega_3 = 1 - \phi_2^2 - \phi_3(\phi_1\phi_2+\phi_3)$$

$$\Omega_4 = \phi_1 + \phi_2(\phi_1+\phi_3) + \phi_1\phi_3(\phi_1+\phi_3) + \phi_2\phi_3$$

$$\lambda = \frac{\phi_1 + \phi_3(\phi_1\phi_2+\phi_3)}{1 - \phi_2-\phi_2\phi_3-\phi_1\phi_3(\phi_1+\phi_3)}$$

$$\mu = \frac{\phi_1\phi_3\theta_2 + \theta_1 - \theta_2(\phi_1-\theta_1)}{1 - \phi_2-\phi_2\phi_3 - \phi_1\phi_3(\phi_1+\phi_3)}$$

As can be seen, the above equations are nonlinear. In order to avoid involved computations, several initial estimates were guessed. The one set which is given in Table 9 leads to the solution.

c. Final estimates of parameters The above guessed initial estimates were used to compute the final estimates of parameters by regression analysis based on minimizing the sum of squares of residuals. The final estimates of parameters with their 95% confidence interval are given in Table 10. Therefore the model is

$$x_t - .98558\, x_{t-1} + .21353\, x_{t-2} - .14157\, x_{t-3} = a_t + 1.132\, a_{t-1} + .2842\, a_{t-2}$$

In the above equation all the roots on each side of the equation are greater than one. Therefore the process is stationary and invertible.

d. Diagnostic checking To assure the adequacy of the model the following two checks were performed:

(1) The autocorrelations and partial autocorrelations, given in Table 11 and Table 12, and their corresponding plots shown in Fig. 9 and Fig. 10, were observed to be statistically insignificant at all 24 lags.

(2) The χ^2 value based on 24 autocorrelations with 19 degrees of freedom is 21.8 and the χ^2 value from the χ^2 table with 19 degrees of freedom at .025 level is 32.9.

The above two checks indicate that the fitted model is adequate.

V. TRANSFER FUNCTION-NOISE MODEL - APPLICATION

1. Transfer Function Model

a. Identification Since the input series is white-noise, it is not necessary to prewhiten it. The cross correlations between the input and the response calculated up to 24 lags were used to compute the impulse response weights also up to 24 lags by applying equation (15). These impulse response weights are shown in Table 13 and plotted in Fig. 11. A dotted line at a distance equal to twice the standard error has been drawn in Fig. 11 to find the number of impulse response weights which are statistically insignificant from the left end of the plot. No particular shape is identifiable in this plot. A guess of the order of the model was made on the basis of previous knowledge of the univariate model of the response series. The guessed order of the transfer function is (3,2). The value of the lag factor b is 3, since the first three impulse response weights are statistically insignificant. So the identified order of the model is (3,2,3).

b. Estimation of Parameters The initial estimates of transfer function parameters, computed with equations (14), are as follows:

$$.287 = \omega_0$$

$$.442 = .287\delta_1 - \omega_1$$

$$.358 = .442\delta_1 + .287\delta_2$$

$$.284 = .358\delta_1 + .442\delta_2 + .287\delta_3$$

$$.254 = .284\delta_1 + .358\delta_2 + .442\delta_3$$

Solving these equations gives the following estimates for the parameters:

$$\delta_1 = .951 \qquad \delta_2 = -.218 \qquad \delta_3 = .140$$

$$\omega_0 = .287 \qquad \omega_1 = -.169$$

2. Noise Model

a. Identification The noise series was generated using equation (19). The autocorrelations of the noise series up to 24 lags and the standard errors of the autocorrelations are given in Table 14, and the plot of autocorrelations is shown in Fig. 12. Partial autocorrelations also estimated up to 24 lags are shown in Table 15 and plotted in Fig. 13. The approximate standard error of partial autocorrelations is .05. The decay of autocorrelations is close to exponential in Fig. 12. In Fig. 13 the partial autocorrelations appear to have a cutoff after 1 but those at lags 5 and 8 are not insignificant. No particular model can be identified [4]. Univariate models AR(2) and ARMA(1,1) were taken as a first guess.

b. Initial Estimates of Parameters The initial estimates of parameters of both AR(2) and ARMA(1,1) identified as noise models were computed. Using equation (4) for AR(2) we obtain

$$0.982 = \phi_1 - \phi_2(0.967)$$

$$0.967 = \phi_1(0.982) - \phi_2$$

from which

$$\phi_1 = .908 \qquad \phi_2 = .075$$

From equation (6) for $k = 0$ and using equation (5), and from equation (6) for $k = 1$ we obtain

$$\rho_1 = \frac{(1-\phi_1\theta_1)(\phi_1-\theta_1)}{1 + \theta_1^2 - 2\phi_1\theta_1}$$

From equation (4) for $k = 2$ we obtain

$$\rho_2 = \phi_1\rho_1$$

Using the above two equations [5] for ARMA(1,1)

$$.982 = \frac{(1-\phi_1\theta_1)(\phi_1-\theta_1)}{1+\theta_1^2-2\phi_1\theta_1}$$

$$.967 = .982\phi_1$$

from which

$$\phi_1 = .984 \qquad \theta_1 = -.101$$

3. Transfer Function-Noise Model

a. Final Estimates of Parameters The initial estimates of transfer function and noise models obtained previously and listed in Table 16 were used to compute the final estimates of the transfer function-noise model by regression analysis based on minimizing the sum of the squares of residuals. For the transfer function-noise model with ARMA(1,1) as the noise model, the final estimates of the parameters and their 95% confidence intervals are given in Table 17. The other transfer function-noise model with AR(2) as the noise model was found inadequate. At the same time the autocorrelations and partial autocorrelations both up to 24 lags and the standard errors of autocorrelations of the residuals were also computed. The autocorrelations and their standard errors up to 24 lags are shown in Table 18 and plotted in Fig. 14. Partial autocorrelations are shown in Table 19 and plotted in Fig. 15. In addition the cross-correlations between prewhitened input and the residuals were also computed and are shown in Table 20. The standard error of cross-correlations, also approximated as $1/\sqrt{n}$, is approximately equal to .05.

b. Fitted model Finally, the equation for the fitted model is obtained as

$$y_t - .84679\, y_{t-1} + .11578\, y_{t-2} - .14304\, y_{t-3} = .23697\, x_{t-3}$$
$$+ .18946\, x_{t-4} + (1-.98285B)^{-1}(1+.65979B)a_t$$

or

$$y_t - 1.82964\, y_{t-1} + .94805\, y_{t-2} - .25683\, y_{t-3} + .14059\, y_{t-4}$$

$$= .23697\, x_{t-3} - .04344\, x_{t-4} - .18621\, x_{t-5} + a_t + .65979\, a_{t-1}$$

4. <u>Diagnostic Checking</u>

To assure the adequacy of the transfer function-noise model, the following four checks were performed.

(1) The autocorrelations and partial autocorrelations of the residuals at all 24 lags shown in Figs. 14 and 15, respectively, were observed to be statistically insignificant.

(2) The χ^2 value based on 24 autocorrelations was computed as 24.1 using equation (7) and the χ^2 value is found to be 36.8 from the table with 22 degrees of freedom at .025 level.

(3) The cross-correlations between the prewhitened input and the residuals at all 24 lags are shown in Table 20 were observed to be statistically insignificant.

(4) The χ^2 value based on 25 cross-correlations was computed as 23.07 using equation (20), and the χ^2 value was found to be 34.2 from the table with 20 degrees of freedom at .025 level.

The above checks indicate that the fitted transfer function-noise model (3,2,3) is adequate.

ACKNOWLEDGMENT

This research was supported by the U. S. Office of Naval Research under Contract N00014-76-C-0825, Project NR064-576, with the University of Wisconsin-Madison.

REFERENCE

1. Box G. E. P. and Jenkins, G. M., Time Series Analysis, Forecasting and Control, Holden-Day, 1970.
2. ibid p. 74
3. ibid p. 76
4. ibid p. 78

Table 1.´ Identifiable Shapes of ACF and PACF

	Damped Out Group	Cutoff Group
1	Decaying exponential	Cutoff after 1
2	Damped sine wave	Cutoff after 2
3	Mixture of two decaying exponentials	Cutoff after 2

Table 2. Shapes of ACF and PACF of AR and MA Models of Orders 1 and 2

Model	ACF	PACF
AR(1)	Decaying exponential	Cutoff after 1
MA(1)	Cutoff after 1	Decaying exponential
AR(2)	Damped sine wave or mixture of two decaying exponentials	Cutoff after 2
MA(2)	Cutoff after 2	Dampled sine wave or mixture of two decaying exponentials

Table 3 Normally Distributed Random Numbers
- White Noise Input

THE INPUT SERIES

-1.046546	.221081	-1.220517	-.062617	.419244	.517110	.207772	3.190364
-1.372380	-.525437	-1.566574	.400818	.601631	-.149277	-.531703	-.737412
1.222771	2.995062	-1.071704	.447527	.486597	.225984	-1.856653	-.492462
.002612	.105822	.278457	-.721910	-1.310037	.931443	.649336	-.219367
-.681643	1.747159	-.133825	2.478115	-1.052217	.415038	-.982722	1.169277
-1.405111	.321967	-.790251	-1.406858	.529309	.385378	-1.264836	-.174614
-1.535732	-.057828	-1.173841	.338153	.153493	-.497639	.309967	-.164509
-.983330	1.612114	1.319080	.029119	.570783	-1.948571	.407847	-.908672
-1.186685	-.501218	-.975144	-.816237	-.093542	1.920960	-.833932	.469804
1.025974	.188285	-1.348076	-.906177	.310981	-.701083	.552288	-1.397470
-.377943	-1.016304	.403281	1.179437	-1.186292	.248971	-.497587	1.487987
-.236139	.058035	1.439927	.202227	-.983803	-.443505	-.290767	-1.309843
-1.601100	-.941135	-.430593	-.376879	.033546	.661411	1.563016	2.012849
-.392953	1.096481	-.602868	.004302	1.501597	1.365850	.153138	-.272796
.828836	-1.600185	1.358568	.703734	.577659	1.057526	.145625	-1.768834
-1.947486	.170427	-.977069	-.334294	-.967266	-.231678	.047810	-.611720
-.745205	.180846	-.961092	1.507648	-.803578	-.014031	-.488516	.211016
-2.261000	.225527	-.275420	-.352027	-1.771070	-.008921	.042778	.161614
-.854142	-.962136	-2.213133	-.435148	-.517048	.585949	.781386	-.102295
.194823	-.115663	.243622	.081380	.384950	-.021842	1.932486	-.355421
1.455679	-.723354	-1.034075	-1.083040	.124866	1.131668	-1.057096	.519001
-1.485437	-.392869	.392745	-1.884397	.074221	.435253	.235725	1.451772
-.021846	1.830793	.129945	-.494824	.862106	.818285	-.488821	-.261612
.215041	-1.366999	1.424756	.480091	-.162888	.324680	.165318	-.314863
-1.324665	-.284668	.611167	2.518418	-2.385473	-.545316	-1.254997	1.400122
-1.292815	.431176	-.328516	-2.054695	.916340	.818566	-.816758	-.706588
.283985	3.217972	-.946325	.166830	-1.248310	-.544560	.715443	.642469
1.392524	.228923	.072160	-2.241863	.778013	-.601280	-.218413	.244239
-.910975	-.159615	-.009383	.347327	.784512	-1.977219	-.642408	.792880
-2.041271	-2.209217	.624794	-1.229084	.279338	-1.538204	-.512677	-.532991
.441235	1.261802	.914292	1.462106	-1.447334	-2.508337	.511058	.457034
1.269315	.145671	-.362281	1.550978	-.902560	-1.300198	2.331706	2.031753
-.217407	.092385	-.508575	-.335826	-.149413	-.128590	-1.203881	.149891
1.024560	-.508623	-2.353647	-.129360	-.861375	.051400	-.406976	.781941
-.292655	-1.634627	-.595100	-.558171	.030908	-1.236767	.150707	-.535260
1.002519	.458075	1.139522	.628067	-1.232769	.247055	-.721322	.542069
.471989	1.708597	.663027	1.046897	-.079510	-1.154816	-.765337	-.006592
.970652	-.113213	.212625	-1.298591	-1.384673	.249679	-.878330	.249074
-.716047	-.314443	-.515466	.043463	.358157	1.088251	.850927	-1.142201
.461123	-.047895	1.129200	-.562863	.012492	-1.457430	.440508	2.500768
-.715913	-.883214	.456815	.477152	.829269	.869519	1.242504	-.432897
-.483919	-2.094217	-.794957	.695382	.787646	.713429	-.506425	-1.646217
-.816020	-.189206	-.241423	.182079	2.131296	-.504185	-.120386	-.803267
-1.219898	-.356627	1.489502	1.028995	.437677	-.609051	1.264824	-1.217023
1.624339	1.508373	1.187668	-.799501	.217363	-.699820	-.647126	.195685
-.964186	-1.353808	-.530056	.417669	.580245	.227057	-.848685	-1.479222
.112529	-.449190	.214705	.455958	.236913	-.499035	-.490270	-.255954
-.212499	-.137747	.745819	1.300966	1.076571	.185054	-.655627	1.439015
1.254740	-.608740	.797991	-.401811	-1.631252	-.521834	-.559468	-.650567
-.078064	1.031868	-.802114	1.040559	-.560647	-1.006463	-.946873	-.273291
.277431	-.456525	.782311	.531742	.150945	1.005575	.055225	.279342
.599951	-.087023	1.317954	-.660961	.141367	.658650	-.012045	-1.425726
.848500	.659264	-.376291	2.161617	.535428	-1.053767	-.914700	.525694
.880612	-.639476	-.050963	.888746	-.428594	-1.626727	-1.508158	-1.276869
.346235	-.903109	-.546014	2.621722	-.630806	-.484731	-1.629762	.868658
-.113989	.062959	.078903	-.687778	.284798	-1.967381	-.124529	-1.023695
.246326	.020254	.339447	.194477	1.171626	-.157970	.650401	-.399325
.149448	.388739	-1.129177	2.220796	-.485001	-.206734	1.661626	1.225031
-.454523	.945280	.033364	.695612	.406317	.115773	-2.532266	-.198552
-.564072	-.523411	1.226922	-.248987	-1.449371	-1.474013	.095373	.813966
1.303209	-1.195834	-1.656535	.247365	-.695266	.410157	1.051289	-2.037500
.153352	-.074327	1.008951	.072706	-1.305213	.598454	1.086163	.534422

Table 4. Autocorrelation Function of Input Series

Mean of the series = -.056198
Standard deviation of series = .98367
Number of observations = 496

Lag	ACF	St Er	Lag	ACF	St Er
1	.02	.04	13	-.03	.05
2	.00	.04	14	.03	.05
3	-.08	.04	15	-.01	.05
4	-.01	.05	16	.03	.05
5	-.02	.05	17	-.08	.05
6	.05	.05	18	-.00	.05
7	.05	.05	19	-.11	.05
8	-.03	.05	20	-.06	.05
9	-.01	.05	21	.00	.05
10	-.07	.05	22	-.03	.05
11	-.10	.05	23	.00	.05
12	-.01	.05	24	-.05	.05

Table 5. Partial Autocorrelation Function of Input Series

Standard error of all partial autocorrelations = .05

Lag	PACF	Lag	PACF
1	.01	13	-.05
2	.00	14	.02
3	-.08	15	-.02
4	-.01	16	.02
5	-.02	17	-.06
6	.05	18	.00
7	.05	19	-.11
8	-.03	20	-.08
9	.00	21	-.02
10	-.06	22	-.07
11	-.10	23	-.02
12	-.01	24	-.06

Table 6 Digitized Response

THE RESPONSE SERIES

-2.588975	-2.108417	-1.594259	-1.705228	-1.995290	-2.304462	-2.426837	-2.155579
-1.784448	-1.262892	-.323352	-.281122	-1.033247	-1.623542	-1.617686	-1.266283
-1.240390	-1.541240	-1.715400	-1.045886	.111894	.290986	.073979	.266326
.199436	-.507993	-1.049276	-1.030781	-.890220	-.848607	-1.139285	-1.566517
-1.344270	-.857238	-.858471	-.974372	-.490731	.073979	.620503	.660576
.354793	.180017	.217007	.009863	-.256771	-.542825	-1.045577	-1.040029
-.808843	-1.123564	-1.524287	-1.924084	-2.189794	-2.367345	-2.269014	-1.997756
-1.943504	-1.800169	-1.722490	-1.741601	-1.105069	-.193888	.138095	.078911
-.367740	-.616804	-.808843	-1.277380	-1.593026	-1.848564	-2.058480	-1.790613
-1.010745	-.685544	-.518165	.049011	.282355	-.165221	-.562244	-.478709
-.370514	-.330750	-.549915	-.882822	-1.001190	-.682461	-.118675	-.075829
-.149500	.072438	.581048	.845833	.771545	1.220354	1.513806	1.222820
.938307	.784183	.456207	-.088467	-.450042	-.503677	-.426615	-.176626
.378220	1.124181	1.833151	2.147872	2.239114	2.259766	2.264698	2.766526
3.378399	3.491834	3.354663	3.296713	3.052889	3.130567	3.555641	3.809628
4.008148	3.886700	3.067685	2.101019	1.738211	1.548330	1.259810	.977763
.808843	.818399	.689551	.423533	.384694	.409358	.687085	.771545
.598926	.510459	.394557	-.019727	-.296226	-.158131	-.342772	-.848299
-.994716	-.710512	-.537898	-.764455	-1.302040	-2.031971	-2.333129	-2.165443
-1.648819	-1.029548	-.696024	-.513233	-.315954	-.081685	.152891	.397948
.669823	1.025541	1.414550	1.657450	1.578230	1.043111	.604166	.692633
1.096130	1.067463	.892686	.595535	.337532	.450042	.241666	.049011
.408737	.778943	1.079793	1.464795	1.952751	2.192260	2.051699	2.194110
2.505132	2.379058	2.089922	1.933023	1.666389	1.846714	2.204282	2.179930
2.133076	2.115815	1.831302	1.267208	.985777	1.380643	2.006078	1.548638
.720376	.426307	.673214	.680612	.450042	.321811	-.155357	-.145493
.345238	.177551	-.200053	.127923	1.084725	1.203400	.632525	.291911
.065040	.288520	.721300	1.072087	1.212956	.936766	.268792	.054251
.241666	.053943	-.029900	-.282355	-.498129	-.458673	-.248756	-.118675
-.710203	-1.203400	-1.203400	-1.785373	-2.781939	-2.925582	-2.863008	-2.988773
-3.304727	-3.580918	-3.521734	-3.063986	-2.238806	-1.392356	-.832578	-1.257652
-2.262541	-2.313093	-1.620460	-.947246	-.633450	-.540667	-.157823	-.236734
-.587213	.157515	1.217888	1.203400	.855080	.699107	.496896	.384077
.296226	.049319	-.010172	.382228	.221939	-.651020	-1.138977	-1.227135
-1.249021	-1.184905	-.897618	-.907482	-1.469727	-1.885245	-1.935798	-1.919152
-2.099477	-2.130919	-1.928708	-1.430579	-.801753	-.180325	.176010	-.044387
-.221939	-.217007	-.015104	.455899	1.003964	1.348277	1.646045	1.585628
1.085341	.647321	.664583	1.014753	1.159013	.991017	.510459	.034523
-.022810	-.071821	-.128539	-.231803	-.405654	-.512925	-.428157	-.069355
.464530	.816241	.586904	.423225	.660576	.934300	.927210	.631292
.256462	.413977	1.208024	1.309438	.757057	.757057	1.109385	1.458321
1.802018	2.130610	1.993748	1.410235	.583822	.100181	.413977	.891762
1.119249	.810692	.123299	-.313180	-.404421	-.385002	-.110353	.557004
.741645	.389626	-.027434	-.513233	-.622353	-.012021	.646088	.784183
.581664	.615263	.606940	.879123	1.572990	1.957992	1.677178	1.321768
.996258	.616804	.483025	.268792	-.372364	-.720067	-.528029	-.197587
-.143027	-.581972	-1.232992	-1.446608	-1.414859	-1.292792	-1.014753	-.850765
-1.014444	-1.292484	-1.420715	-1.450615	-1.385883	-.943547	-.207451	.373597
.453741	.288520	.570875	1.024925	.939540	.789115	.631292	.061341
-.438020	-.690476	-.942006	-.949404	-.642697	-.582281	-.490731	-.567176
-1.104761	-1.605664	-1.706769	-1.568674	-1.484523	-1.188604	-.780792	-.513233
-.166762	.049319	.160289	.337532	.483025	.688318	.651020	.493197
.620195	.537276	.092474	.117134	.496896	.631292	.946630	1.274606
.789115	.217007	.289753	.645780	.500595	.330750	.498129	.368356
-.364041	-1.292484	-1.847022	-1.923468	-2.076359	-2.054473	-1.233300	-.887754
-1.385883	-1.903740	-1.785681	-1.474967	-1.417016	-1.400679	-1.496852	-1.652210
-2.131227	-2.513455	-2.613943	-2.513146	-2.209830	-1.900041	-1.504867	-.995025
-.680612	-.498437	-.435246	-.364041	-.266326	-.264169	.196970	.467921
.394557	.753666	1.306972	1.327624	1.358757	1.538466	1.689199	1.803559
1.541240	.691709	.141178	.221939	.149500	.421375	.475935	-.172619
-.853231	-.853231	-.276498	.215465	-.093707	-.843367	-1.019068	-.957110
-.711436	-.399489	-.758290	-1.052975	-.782025	-.315954	-.167687	-.464838

Table 7. Autocorrelation Function of Response to White Noise

Mean of the series = -.06102
Standard deviation of series = 1.2854
Number of observations = 496

Lag	ACF	St Er	Lag	ACF	St Er	Lag	ACF	St Er	Lag	ACF	St Er
1	.95	.04	13	.23	.14	25	.04	.15	37	-.01	.15
2	.87	.08	14	.19	.14	26	.05	.15	38	-.01	.15
3	.79	.09	15	.15	.14	27	.06	.15	39	-.01	.15
4	.73	.11	16	.12	.14	28	.06	.15	40	-.01	.15
5	.67	.12	17	.08	.15	29	.05	.15	41	-.02	.15
6	.61	.12	18	.05	.15	30	.05	.15	42	-.04	.15
7	.55	.13	19	.03	.15	31	.05	.15	43	-.05	.15
8	.48	.13	20	.01	.15	32	.05	.15	44	-.06	.15
9	.41	.14	21	.01	.15	33	.04	.15	45	-.07	.15
10	.34	.14	22	.02	.15	34	.02	.15	46	-.08	.15
11	.30	.14	23	.02	.15	35	.00	.15	47	-.09	.15
12	.26	.14	24	.03	.15	36	-.01	.15	48	-.10	.15

Table 8. Partial Autocorrelations of Response to White Noise

Standard error of all partial autocorrelations ≃ .05

Lag	PACF	Lag	PACF	Lag	PACF	Lag	PACF
1	.95	13	-.01	25	.01	37	-.00
2	-.50	14	-.03	26	-.02	38	.03
3	.36	15	-.02	27	-.07	39	.02
4	-.16	16	-.02	28	-.06	40	-.08
5	.05	17	-.03	29	.02	41	.00
6	-.08	18	-.02	30	.05	42	-.02
7	-.09	19	.02	31	-.02	43	-.01
8	-.04	20	.12	32	-.03	44	-.04
9	-.03	21	.03	33	-.04	45	.00
10	.01	22	-.02	34	-.06	46	-.01
11	.09	23	.07	35	.02	47	.00
12	-.03	24	.00	36	.05	48	.04

Table 9. Guessed Estimate of Parameters

Parameter Number	Parameter Type	Parameter Order	Beginning Value
1	Autoregressive	1	.92
2	Autoregressive	2	-.21
3	Autoregressive	3	.22
4	Mean	0	-.061
5	Moving average	1	.19
6	Moving average	2	-.13

Initial residual sum of squares = 86.75

Table 10. Final Estimates of ARMA(3,2) Model for Response

Parameter Number	Parameter Type	Parameter Order	Estimated Value	95 Percent Lower Limit	95 Percent Upper Limit
1	Autoregressive	1	.98558	0.57209	1.3991
2	Autoregressive	2	-0.21353	-0.7153	0.28823
3	Autoregressive	3	0.14157	-0.011436	0.29457
4	Mean	0	-0.06368	-0.64548	0.51812
5	Moving average	1	-1.13200	-1.5526	-0.71140
6	Moving average	2	-0.28420	-0.67322	0.10483

Final residual sum of squares 26.237

Table 11. Autocorrelation of Residuals and Standard Error

Mean of the series = -0.00134
Standard deviation of series = 0.23092
Number of observations = 493

Lag	ACF	St Er	Lag	ACF	St Er
1	.00	.05	13	-.00	.05
2	.00	.05	14	.03	.05
3	-.03	.05	15	.02	.05
4	.00	.05	16	.03	.05
5	.00	.05	17	-.07	.05
6	.05	.05	18	.01	.05
7	.10	.05	19	-.09	.05
8	-.04	.05	20	-.08	.05
9	.02	.05	21	.02	.05
10	-.06	.05	22	-.03	.05
11	-.07	.05	23	.00	.05
12	-.00	.05	24	.00	.05

Table 12. Partial Autocorrelations of Residuals

Standard error for all partial autocorrelations = .05

Lag	PACF	Lag	PACF
1	.00	13	-.02
2	.00	14	.02
3	-.03	15	-.01
4	.00	16	.03
5	.00	17	-.05
6	.05	18	.02
7	.10	19	-.09
8	-.04	20	-.08
9	.03	21	.01
10	-.06	22	-.04
11	-.07	23	-.00
12	.00	24	-.00

Table 13. Impulse Response Weights

Standard error for all impulse response weights ≈ .05

Lag	Weight	Lag	Weight
0	-.111		
1	-.115	13	.144
2	-.013	14	.091
3	.287	15	.067
4	.442	16	.064
5	.358	17	.061
6	.284	18	.053
7	.254	19	.040
8	.238	20	.014
9	.235	21	.013
10	.243	22	-.046
11	.223	23	-.082
12	.185	24	-.090

Table 14. Autocorrelation Function of Noise

Mean of the series = 0.19121
Standard deviation of series = 0.82270
Number of observations = 492

Lag	ACF	St Er	Lag	ACF	St Er
1	.98	.05	13	.79	.20
2	.97	.08	14	.77	.21
3	.96	.10	15	.75	.22
4	.94	.12	16	.73	.22
5	.93	.13	17	.71	.23
6	.92	.14	18	.69	.23
7	.90	.15	19	.67	.23
8	.88	.17	20	.65	.24
9	.86	.17	21	.63	.24
10	.84	.18	22	.61	.25
11	.82	.19	23	.59	.25
12	.81	.20	24	.57	.25

Table 15. Partial Autocorrelations of Noise

Standard error of all partial autocorrelations ≃ .05

Lag	PACF	Lag	PACF
1	.98	13	.01
2	.06	14	-.08
3	.09	15	.01
4	-.11	16	.00
5	.14	17	-.03
6	-.05	18	-.01
7	-.07	19	-.01
8	-.15	20	.04
9	.02	21	-.04
10	-.10	22	-.03
11	.06	23	.01
12	-.02	24	.00

Table 16. Transfer Function and Noise Models - Initial Estimates

	Parameter Type	Parameter Order	Beginning Value
Transfer Function Parameters	Output lag	1	.92
	Output lag	2	-.21
	Output lag	3	.22
	Input lag	0	.19
	Input lag	1	-.13
Noise Model Parameters	Autoregressive	1	.98
	Mean	0	-.061
	Moving average	1	-.54

Estimation for b = 3
Initial residual sum of squares = 6.766

Table 17. Final Estimates of Transfer Function - Noise Model

Parameter Type	Parameter Order	Estimated Value	95 Percent Lower Limit	95 Percent Upper Limit
	(Transfer dunction parameters)			
Output lag	1	.87679	.79763	.89595
Output lag	2	-.11578	-.18265	-.04890
Output lag	3	.14304	.10028	.18580
Input lag	0	.23697	.22360	.25034
Input lag	1	-.18946	-.20091	-.17802
	(Noise model parameters)			
Autoregressive	1	.98285	.96507	1.00600
Mean	0	.34838	-.53963	1.23600
Moving Average	1	-.65979	-.78014	-.53944

Optimum value of b = 3
Final residual sum of squares = 4.7425

Table 18. Autocorrelation Function of Residuals
- Transfer Function - Noise Model

Mean of the series = 0.00026
Standard deviation of the series = 0.09838
Number of observations = 491

Lag	ACF	St Er	Lag	ACF	St Er
1	.02	.05	13	.06	.05
2	.06	.05	14	.00	.05
3	.03	.05	15	-.01	.05
4	.03	.05	16	-.02	.05
5	.06	.05	17	-.05	.05
6	.03	.05	18	.01	.05
7	.09	.05	19	-.10	.05
8	-.03	.05	20	-.04	.05
9	.04	.05	21	-.01	.05
10	-.05	.05	22	-.04	.05
11	-.05	.05	23	.01	.05
12	-.02	.05	24	-.02	.05

Table 19. Partial Autocorrelations of Residuals
- Transfer Function - Noise Model

Standard error for all partial autocorrelations is .05

Lag	PACF	Lag	PACF
1	.02	13	.07
2	.06	14	.00
3	.03	15	-.00
4	.02	16	-.02
5	.06	17	-.03
6	.02	18	.01
7	.09	19	-.10
8	-.04	20	-.05
9	.03	21	-.00
10	-.06	22	-.04
11	-.06	23	.03
12	-.02	24	.01

Table 20. Cross Correlations of Input White Noise and Estimated Residuals

Standard error of all cross-correlations ≃ .05

Lag	Cross Correlation	Lag	Cross Correlation
0	-.014		
1	.048	13	-.045
2	-.032	14	-.009
3	.069	15	.057
4	-.075	16	-.008
5	.011	17	.006
6	-.029	18	-.031
7	.063	19	-.011
8	-.011	20	.057
9	.028	21	-.053
10	-.051	22	.067
11	-.015	23	-.057
12	-.022	24	.063

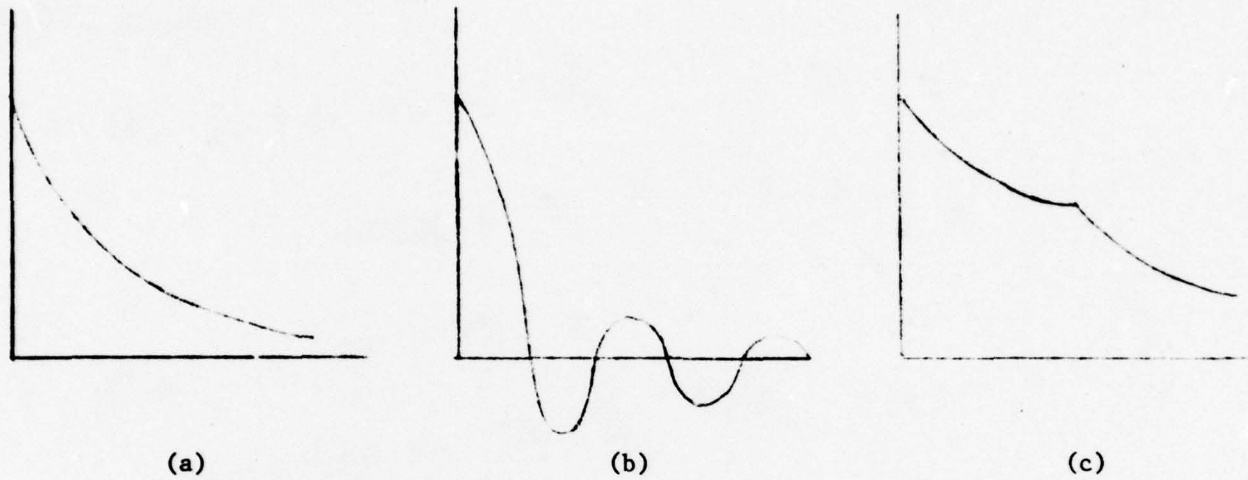

Fig. 1 Three basic damped out ACF and PACF with lag: (a) Decaying exponential, (b) Damped sine wave, and (c) Mixture of two decaying exponentials.

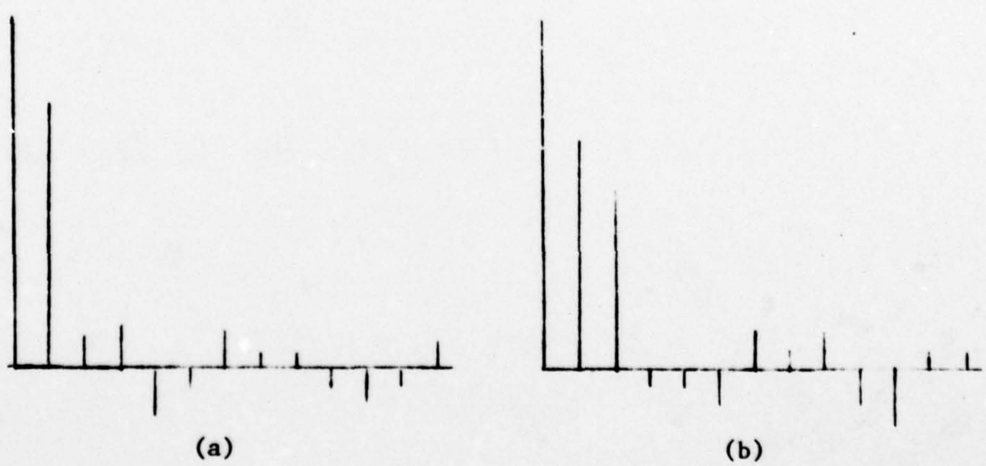

Fig. 2 Two basic cutoffs of the ACF and PACF with lag: (a) Cutoff after 1, and (b) Cutoff after 2.

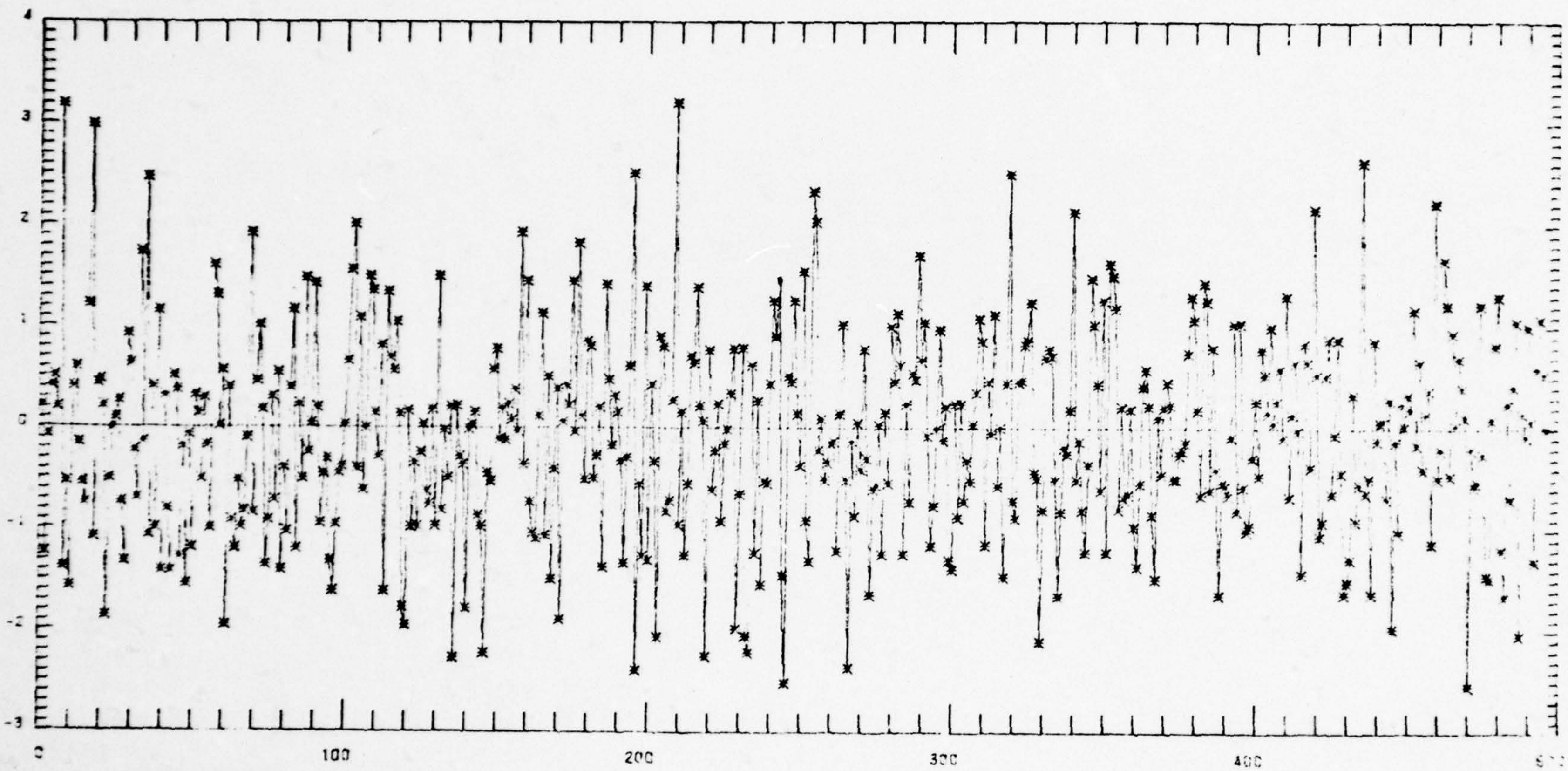

Fig. 3 Plot of Normally Distributed Random Numbers - White Noise Input

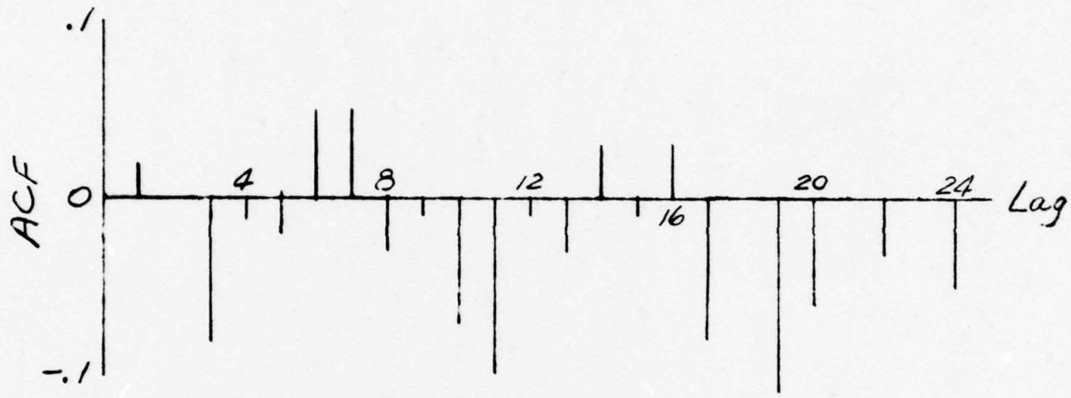

Fig. 4 Autocorrelation Function of Input White Noise

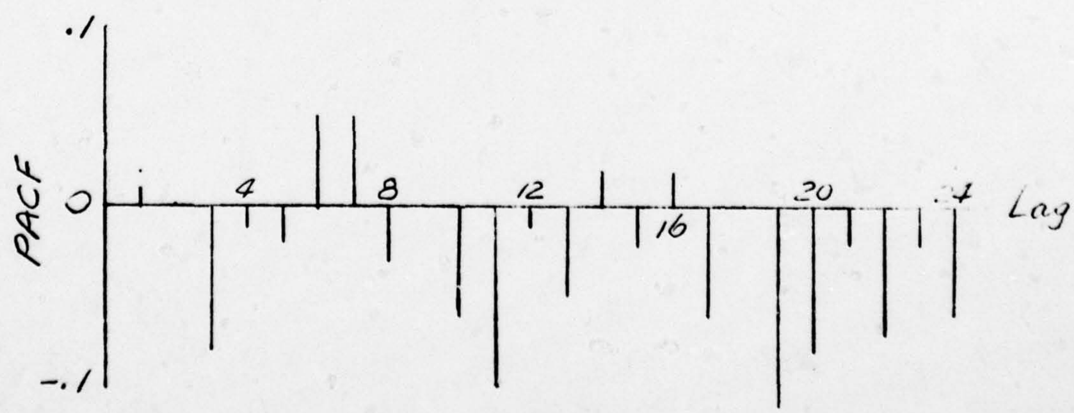

Fig. 5 Partial Autocorrelation Function of Input White Noise

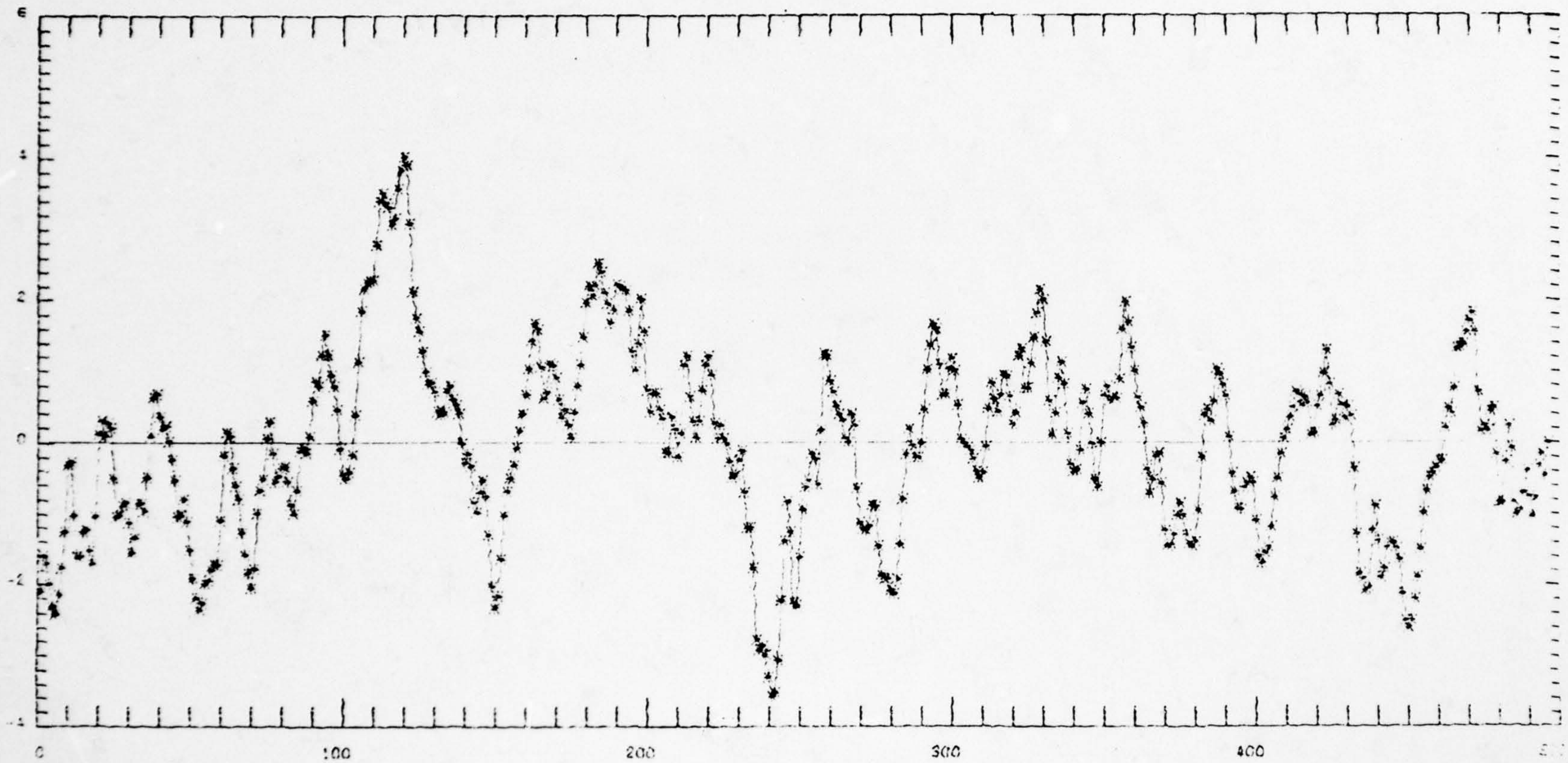

Fig. 6 Plot of Digitized Response

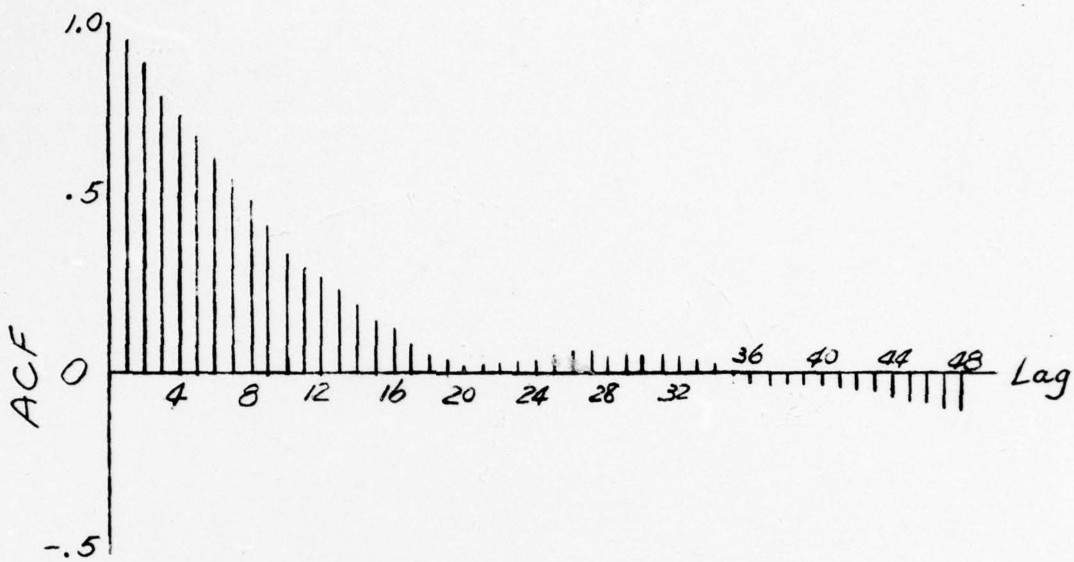

Fig. 7 Autocorrelation Function of Response to White Noise

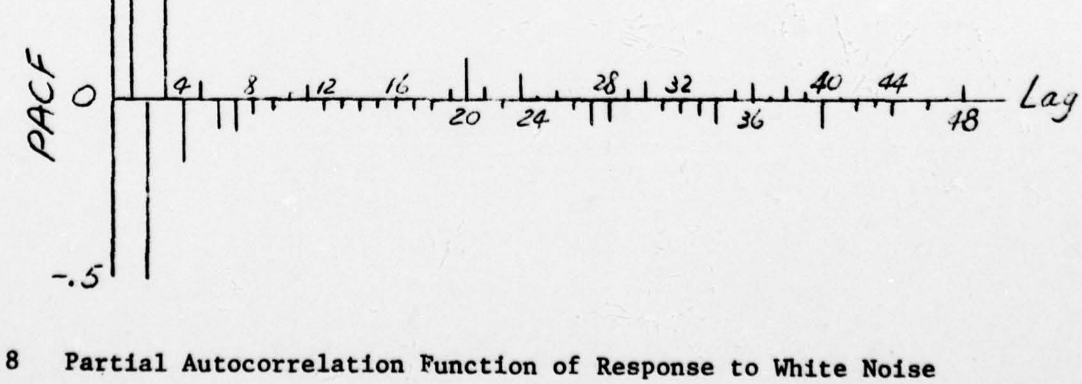

Fig. 8 Partial Autocorrelation Function of Response to White Noise

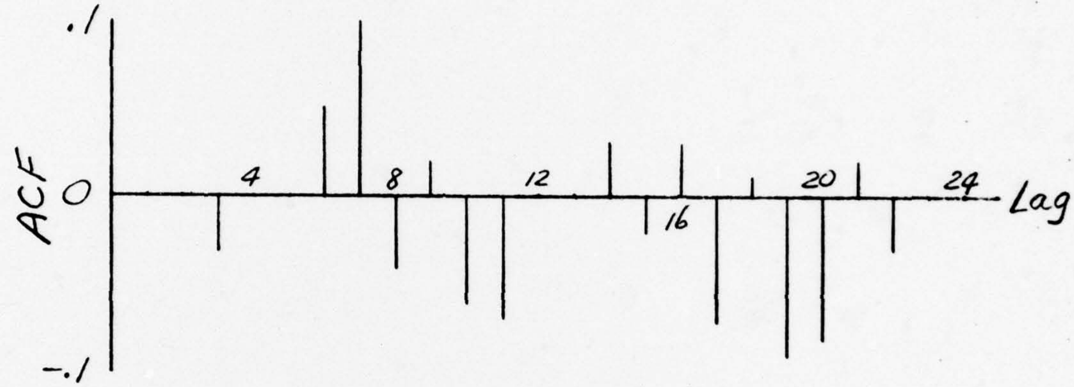

Fig. 9 Autocorrelation Function of Estimated Residuals

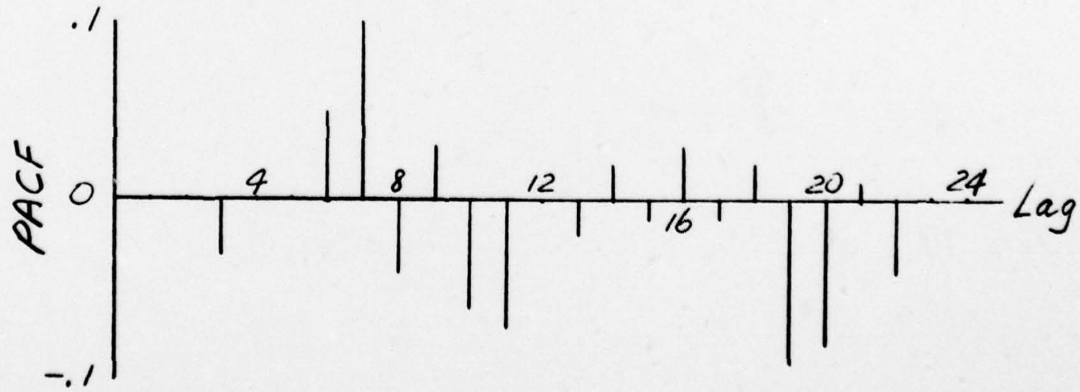

Fig. 10 Partial Autocorrelations of Estimated Residuals

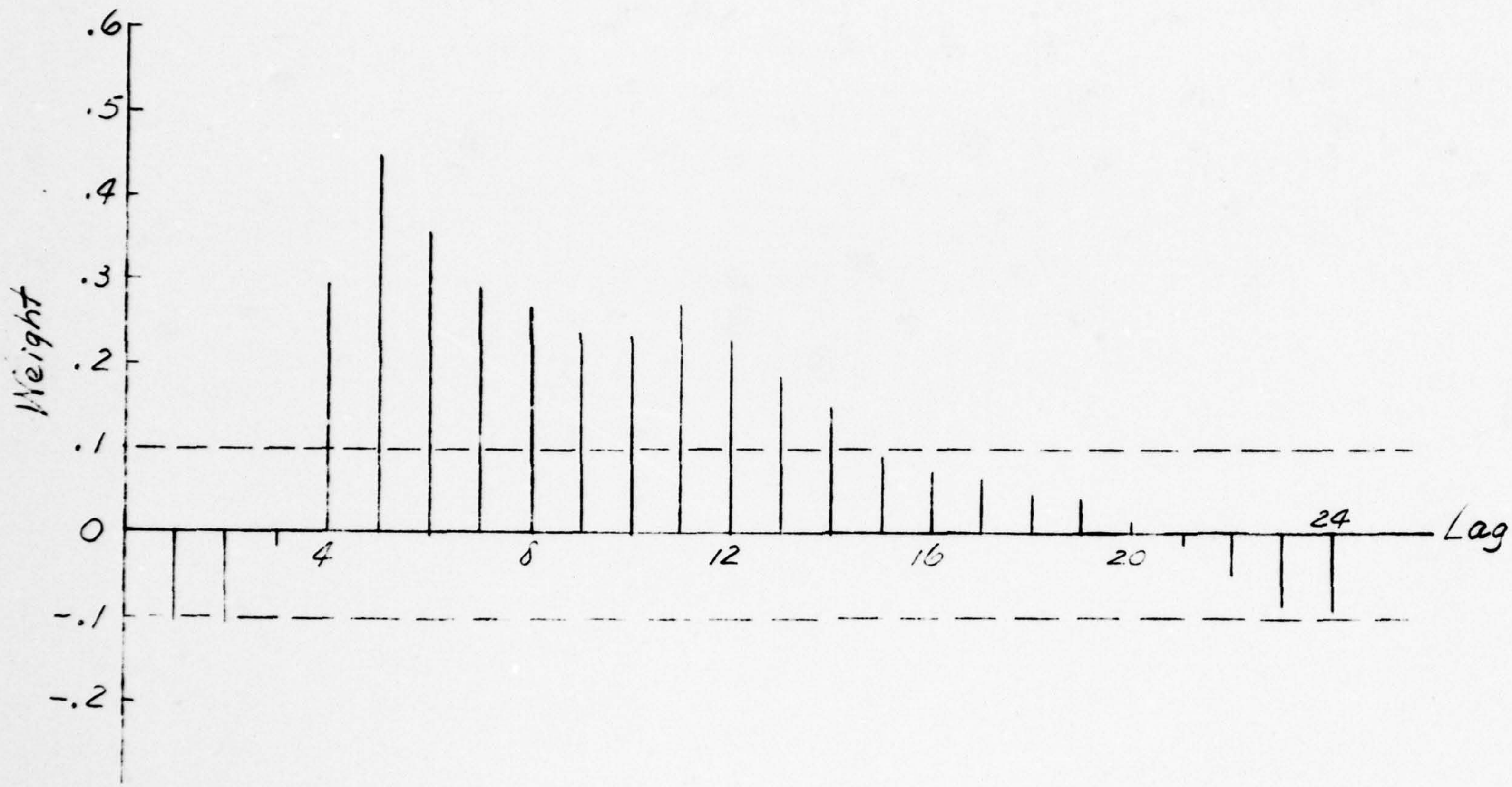

Fig. 11 Impulse Response Weights

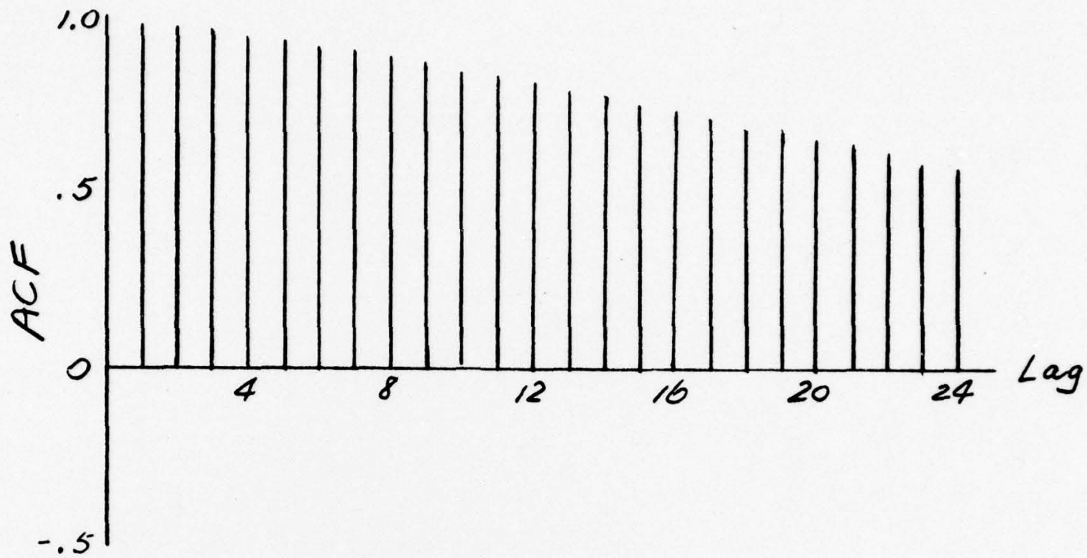

Fig. 12 Autocorrelation Function of Noise

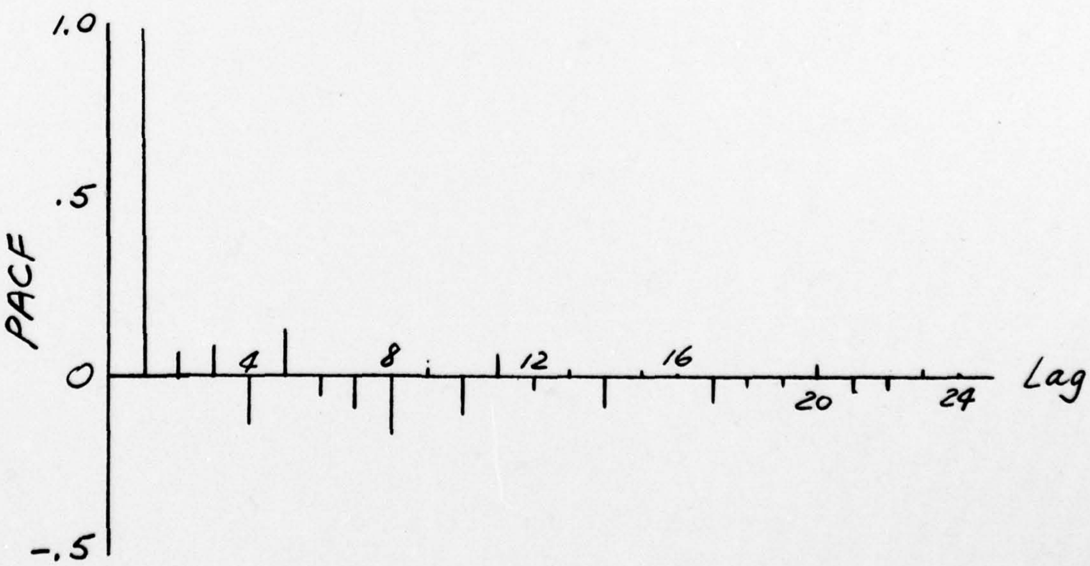

Fig. 13 Partial Autocorrelations of Noise

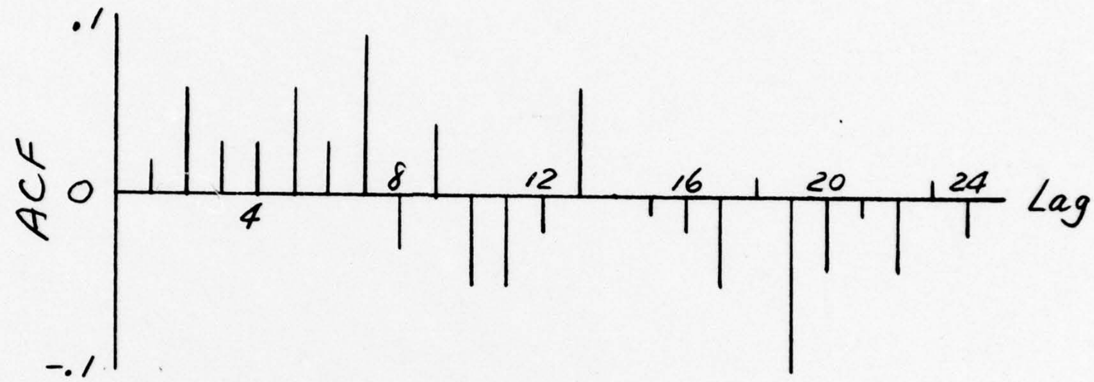

Fig. 14 Autocorrelations of Residuals

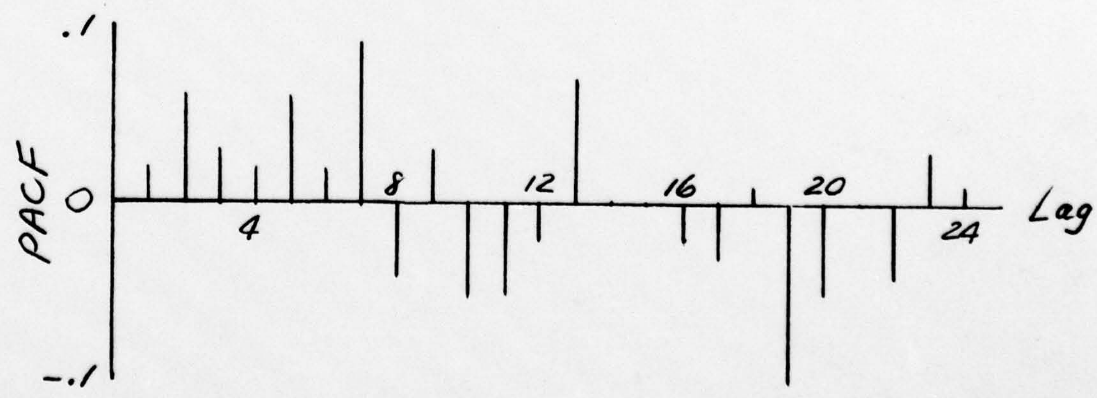

Fig. 15 Partial Autocorrelations of Residuals

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1. REPORT NUMBER: UW/RF-2	2. GOVT ACCESSION NO. / 3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): Time Series Determination of Transfer Functions in Random Fatigue	5. TYPE OF REPORT & PERIOD COVERED: Interim 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): T. C. Huang Vinod K. Nagpal K. S. Shen	8. CONTRACT OR GRANT NUMBER(s): N00014-76-C-0825
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Department of Engineering Mechanics University of Wisconsin Madison, Wisconsin 53706	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: NR064-576
11. CONTROLLING OFFICE NAME AND ADDRESS: Office of Naval Research Department of the Navy Arlington, Virginia 22217	12. REPORT DATE: December, 1977 13. NUMBER OF PAGES: 44
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): Office of Naval Research Chicago Branch Office 536 S. Clark St. Chicago, Illinois 60605	15. SECURITY CLASS. (of this report): Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

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DISTRIBUTION STATEMENT A

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17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Random fatigue
Time series
Transfer function

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Time series determination of the transfer function which relates the input random excitation and the output response in random fatigue experiment is established. This process involves determination of univariate time series of input and output, transfer function and noise models, and the transfer function-noise model.

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